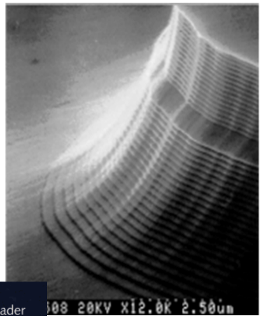
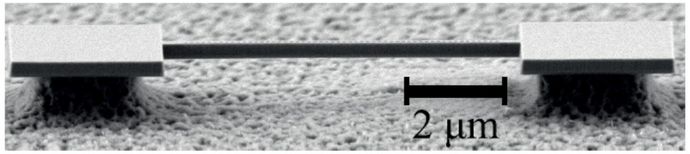
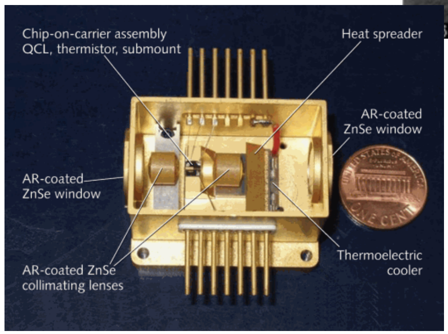
Lecture 14 – 28/05/2025

Laser diodes

- Relaxation resonance and frequency response
- Vertical cavity surface emitting lasers
- High-β nanolasers
- Quantum cascade lasers







Temporal behavior of LDs

The temporal behavior of a laser diode, *when neglecting spontaneous photons*, can be suitably described by the following set of coupled dynamical differential equations:

Pump rate

Carrier recombination rate (ABC-model)

$$\boxed{\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= \frac{J}{qd} - \frac{n}{\tau} - c' g(n) s (n - n_{\mathrm{tr}}) \\ \frac{\mathrm{d}s}{\mathrm{d}t} &= c' g(n) s (n - n_{\mathrm{tr}}) \Gamma - \frac{s}{\tau_{\mathrm{cav}}}\end{aligned}}$$

Conversion of free carriers into stimulated photons

To be seen during the series!

Total stimulated photon loss

Effective overlap with gain medium

$$\boxed{\text{We recall that } \frac{1}{\tau_{\mathrm{cav}}} = \frac{1}{\tau_{\mathrm{i}}} + \frac{1}{\tau_{\mathrm{m}}} = c'(\alpha_{\mathrm{i}} + \alpha_{\mathrm{m}})}$$

$c' = \dfrac{c}{n_{\mathrm{sc}}}$ group velocity of photons

$g(n)(n - n_{\mathrm{tr}})$ optical gain of the medium with $g(n)$ the dynamical variation of the optical gain

s stimulated photon density in the cavity

Temporal behavior of LDs

Steady-state solution to the system of coupled equations:

$$\frac{\mathrm{d}n}{\mathrm{d}t} = 0 \quad \Rightarrow \frac{J_0}{qd} - \frac{n_0}{\tau} - c'gs_0\left(n_0 - n_{\mathrm{tr}}\right) = 0$$

$$\frac{\mathrm{d}s}{\mathrm{d}t} = 0 \quad \Rightarrow c'gs_0\left(n_0 - n_{\mathrm{tr}}\right)\Gamma - \frac{s_0}{\tau_{\mathrm{cav}}} = 0$$

$$\Rightarrow s_0 = \Gamma\tau_{\mathrm{cav}}\left(\frac{J_0}{qd} - \frac{n_0}{\tau}\right) \quad \text{and} \quad \left(n_0 - n_{\mathrm{tr}}\right) = \frac{1}{c'g\Gamma\tau_{\mathrm{cav}}}$$

$$\Rightarrow \boxed{s_0 = \Gamma\tau_{\mathrm{cav}}\left(\frac{J_0}{qd} - \frac{n_{\mathrm{tr}}}{\tau} - \frac{1}{c'g\Gamma\tau_{\mathrm{cav}}\tau}\right)}$$

In the steady-state, the photon density in the cavity is a linearly increasing function of the current density while the electron-hole plasma population is clamped!

Temporal behavior of LDs

Direct modulation of the excitation current (small-signal analysis):

DC current AC current

$$J = J_0 + \delta j \quad \left(\text{with } \delta j << J_0\right) \quad \text{and} \quad n = n_0 + \delta n,\, s = s_0 + \delta s$$

AC terms << DC terms

$$\frac{\mathrm{d}\delta n}{\mathrm{d}t} = \frac{\delta j}{qd} - \delta n\left(\frac{1}{\tau} + c'gs_0\right) - c'g\delta s\left(n_0 - n_{\mathrm{tr}}\right)$$

$$\frac{\mathrm{d}\delta s}{\mathrm{d}t} = \underbrace{c'g\delta s\left(n_0 - n_{\mathrm{tr}}\right)\Gamma - \frac{\delta s}{\tau_{\mathrm{cav}}}}_{=0} + c'gs_0\delta n\Gamma$$

2nd order terms are neglected

As $n_0 - n_{\mathrm{tr}} = \dfrac{1}{c'g\Gamma\tau_{\mathrm{cav}}}$, we get:

$$\boxed{\begin{aligned} &\frac{\mathrm{d}\delta n}{\mathrm{d}t} = \frac{\delta j}{qd} - \delta n\left(\frac{1}{\tau} + c'gs_0\right) - \frac{\delta s}{\Gamma\tau_{\mathrm{cav}}} \\ &\text{and} \quad \frac{\mathrm{d}\delta s}{\mathrm{d}t} = c'gs_0\delta n\Gamma \end{aligned}}$$

Temporal behavior of LDs

Small-signal frequency response is derived from harmonic analysis:

$$\delta j = \Re\left(\delta j(\omega)\exp(i\omega t)\right),\ \delta n = \Re\left(\delta n(\omega)\exp(i\omega t)\right),\ \text{and}\ \delta s = \Re\left(\delta s(\omega)\exp(i\omega t)\right)$$

$$\frac{\mathrm{d}^2\delta s}{\mathrm{d}t^2} = -\omega^2\delta s = \Gamma c' g s_0 \frac{\mathrm{d}\delta n}{\mathrm{d}t}$$

$$\boxed{\frac{\delta s(\omega)}{s_0} = \frac{\Gamma c' g(\delta j/qd)}{-\omega^2 + i\omega\left(1/\tau + c' g s_0\right) + \dfrac{c' g s_0}{\tau_{\text{cav}}}}}$$

$$\boxed{\left|\delta s(\omega)\right| = \frac{\left|\delta s(0)\right|}{\dfrac{\tau_{\text{cav}}}{c' g s_0}\left[\left(\omega^2 - \dfrac{c' g s_0}{\tau_{\text{cav}}}\right)^2 + \omega^2\left(1/\tau + c' g s_0\right)^2\right]^{1/2}}}$$

with $\left|\delta s(0)\right| = \Gamma\tau_{\text{cav}}(\delta j/qd)$

Temporal behavior of LDs

Frequency response exhibits a maximum for the oscillation relaxation angular frequency ω_R defined by:

$$\frac{\mathrm{d}\left|\delta s(\omega)\right|}{\mathrm{d}\omega} = 0$$

$$\boxed{\begin{aligned} \omega_{\mathrm{R}} &= \left[\frac{c' g s_0}{\tau_{\mathrm{cav}}} - \frac{1}{2}\left(\frac{1}{\tau} + c' g s_0\right)^2\right]^{1/2} \approx \left[\frac{c' g s_0}{\tau_{\mathrm{cav}}}\right]^{1/2} \\ \nu_{\mathrm{R}} &\approx \frac{1}{2\pi}\left[\frac{c' g s_0}{\tau_{\mathrm{cav}}}\right]^{1/2} \end{aligned}}$$

$\nu_R \approx$ modulation bandwidth of SC LDs

Efficient mean to modulate the output power!

Oscillation relaxation frequency originates from the **energy exchange** between **electron-hole pair population** and **photon population** which are coupled together via the stimulated emission process. When *n* increases, the gain increases together with *s*, which subsequently leads to a more efficient stimulated emission process and thus to a decrease in *n, hence* the presence of oscillations.

The relaxation originates from the loss of photons via parasitic absorption or through the mirrors (≡ the system is not closed. You have a pump and a leak source).

Temporal behavior of LDs

Illustration, oscillation relaxation frequency of a GaAs/AlGaAs heterojunction laser diode

Relevant parameters:

- Output power: P_s = 10 mW
- Photon energy: $h\nu$ = 1.4 eV
- Output coupler reflectivity: R_{m1}= 1, R_{m2}= 0.32
- Optical mode surface: $A = wd_{mode}$ where w is the cavity width (2 μm) and d_{mode} (= d/Γ) is the effective thickness of the mode (0.1 μm), i.e., $A = 2 \times 10^{-9}$ cm^2
- Cavity length: L = 200 μm
- Parasitic losses: α_p = 10 cm^{-1}
- Optical refractive index: n_{SC}= 3.3
- Quantum efficiency: η = 1
- Dynamical gain: $g = 3 \times 10^{-16}$ cm^2

We recall that: $$\boxed{\tau_{cav} = \frac{1}{c'\left[\alpha_p + \frac{1}{2L}\ln\left(\frac{1}{R_{m1}R_{m2}}\right)\right]}}$$

P_s= (photon energy)(density of photons)(effective volume of the mode)(photon escape rate)

= $h\nu$ s_0 Lwd/Γ $c'\alpha_m$

Γ taken equal to 1 for a heterojunction LD!

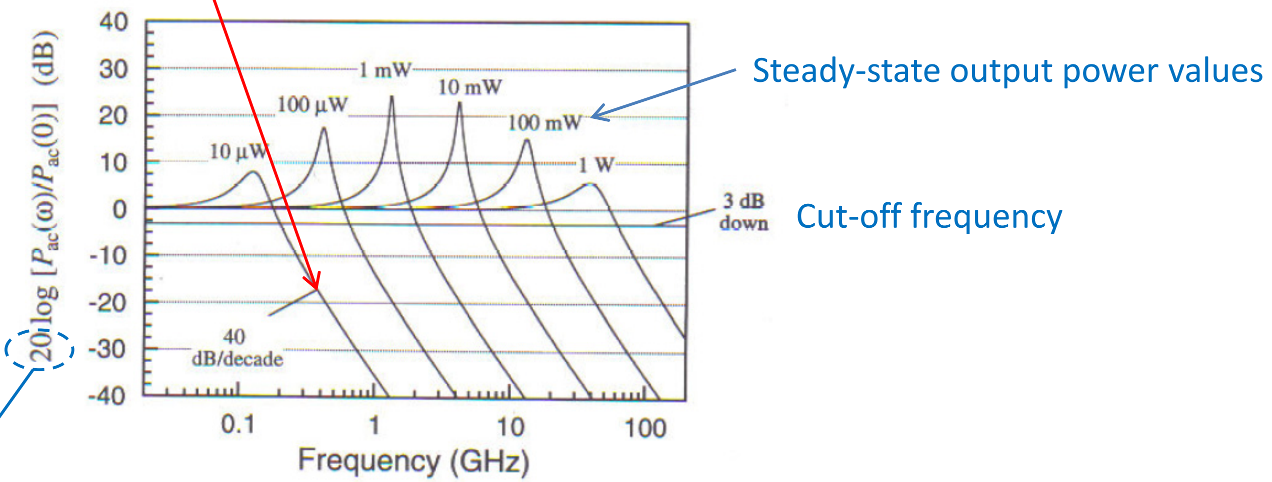
$$s_0 = \frac{P_s}{h\nu}\frac{1}{AL}\frac{n_{sc}}{c}2L\left(\ln\left(1/R_{m_2}\right)\right)^{-1} \approx 4.3\times10^{15}\text{ cm}^{-3} \quad \text{and} \quad \boxed{\nu_R \approx 10.2\text{ GHz}}$$

$$\alpha_m = \frac{1}{2L}\ln\frac{1}{R_1R_2}$$

Temporal behavior of LDs

Main characteristics of $\delta s(\omega)$:

- Flat response at small frequencies ($|\delta s(\omega)| = |\delta s(0)|$) followed by a peak at ω_R and a strong decrease as ω^{-2} (40 dB/decade)
- The expression of ω_R predicts an increase proportional to $(s_0)^{0.5}$

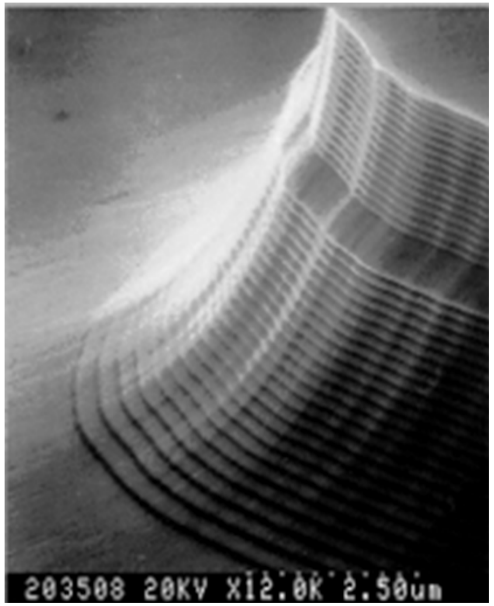


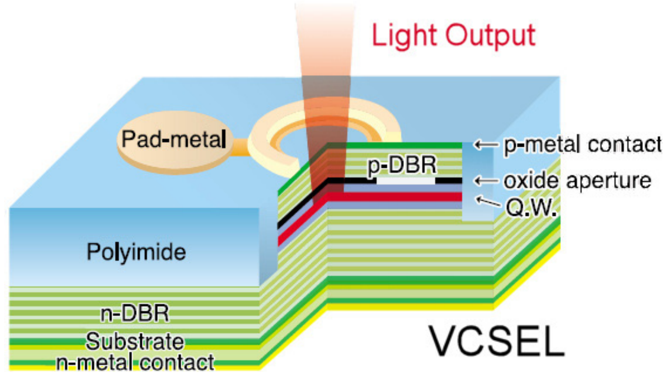
Main application:

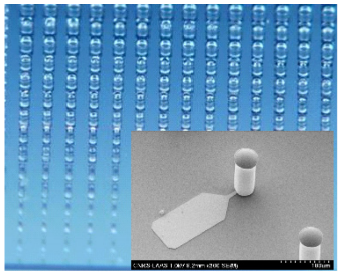
- Semiconductor QW-LDs used as light source for optical telecommunication systems
- The higher the modulation bandwidth, the larger the bit rate (e.g., > 10 Gbit/s with the *basic* non-return to zero, inverted scheme (2 bits per cycle) applied to the laser pictured above)

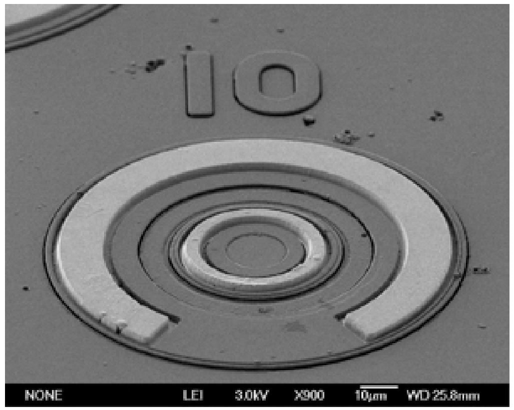
Nota bene: The 20log[] notation is used because photodetection generates an electrical current in direct proportion to the optical power. Thus, for a power ratio in the electrical circuit, this current must be squared.

Vertical cavity surface emitting lasers (VCSELs)









VCSELs

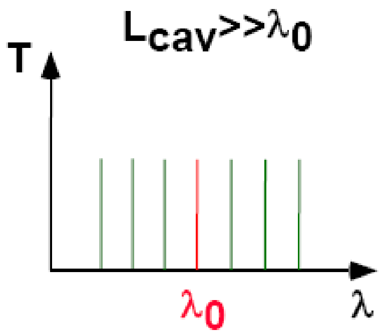
Edge emitting laser diodes

- Multimode
- Beam properties (elliptic shape)
- Large surfaces
- Cannot be tested on wafer

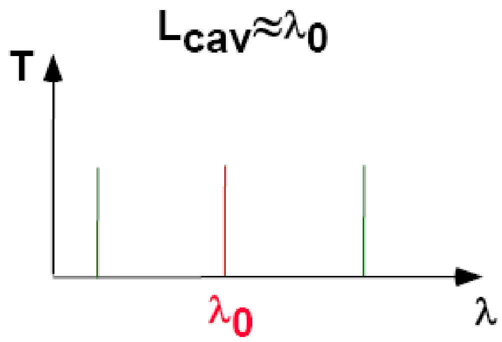
VCSELs

- Monomode (longitudinal)
- Circular beam
- Compact (high density)
- Can be tested on wafer
- Low threshold current

Edge emitting laser



VCSEL



VCSELs

Lasing condition:

$$\Gamma\gamma_{\text{thr}} = \alpha_{\text{i}} + \frac{1}{2L_{\text{eff}}}\ln\left(\frac{1}{R_1R_2}\right)$$

$$\Gamma = \frac{\int_{-d/2}^{d/2}|E(z)|^2 dz}{\int_{-\infty}^{\infty}|E(z)|^2 dz}$$ with $E(z) = E_0\cos(\pi z/L_{\text{eff}})$ $\quad -L_{\text{eff}}/2 < z < L_{\text{eff}}/2$

$$\Gamma = \frac{d\,E_0^2}{\int_{-L_{\text{eff}}/2}^{L_{\text{eff}}/2} E_0^2\cos^2\left(\frac{\pi z}{L_{\text{eff}}}\right)dz} = 2\frac{d}{L_{\text{eff}}}$$

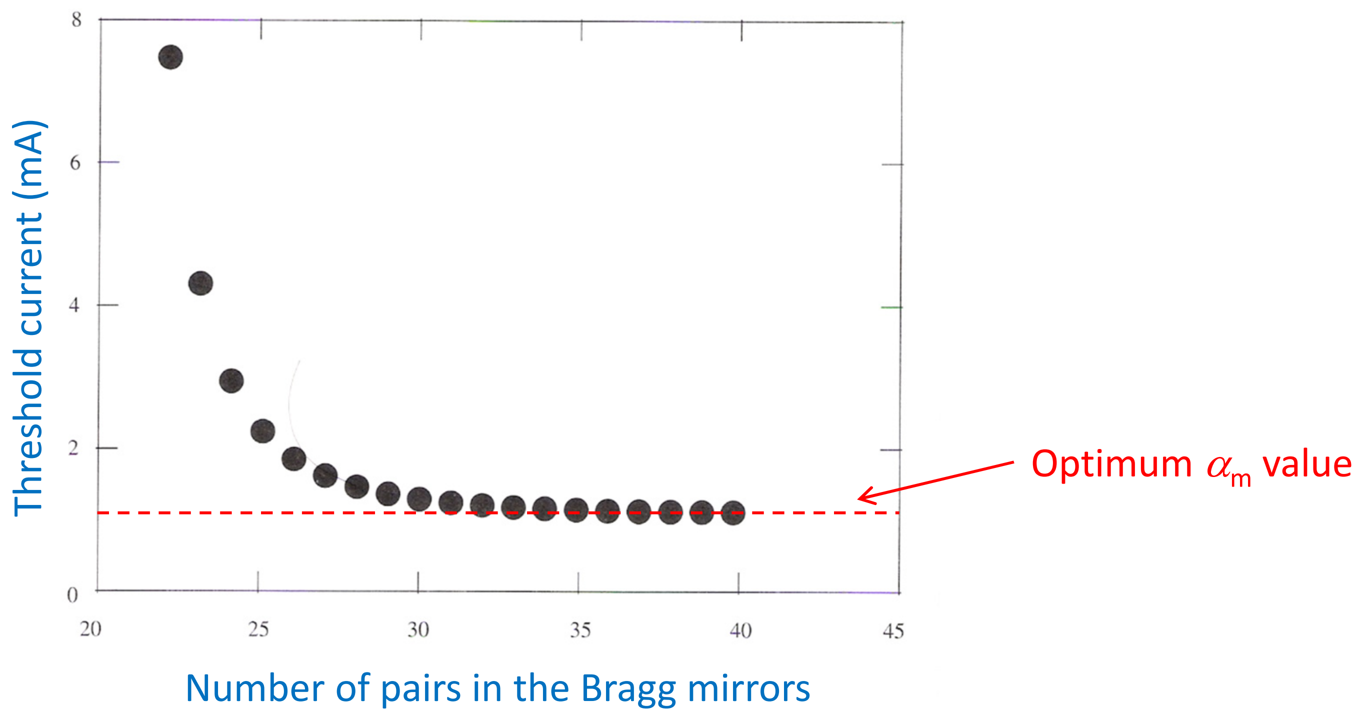
Thickness of the gain medium

Finally,

$$\boxed{\gamma_{\text{thr}} = \frac{1}{4d}\ln\left(\frac{1}{R_1R_2}\right) + \frac{L_{\text{eff}}}{2d}\alpha_{\text{i}}}$$

d is small $\Rightarrow$ R must be very high (> 99%) to minimize the losses!

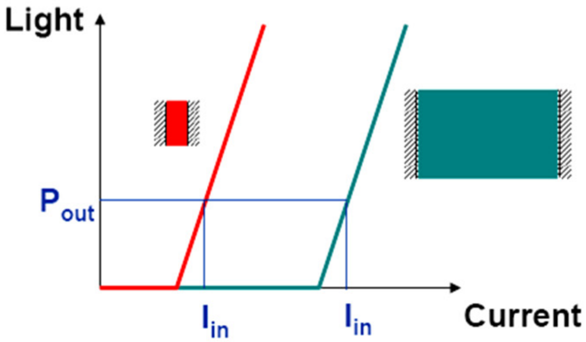
VCSELs



Mirror reflectivity has a strong impact on the threshold current

VCSELs

Threshold current:



Example:

- **Edge-emitting lasers: 5x500 μm² ⇒ I (transparency)=1 mA**
- **VCSELs: 5x5 μm² ⇒ I(transparency)=10 μA**

Beam properties

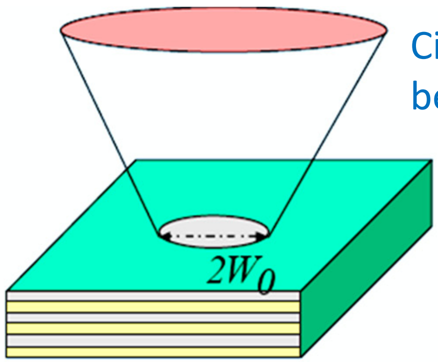
Rayleigh ⇨ $\theta = \dfrac{\lambda}{\pi W_0}$

Diameter $2W_0 = 10\ \mu m$

⇩

$\theta \cong 4^\circ$ @ $\lambda \sim 1\ \mu m$

⇒ small beam divergence



Circular beamshape

Typical features of LDs

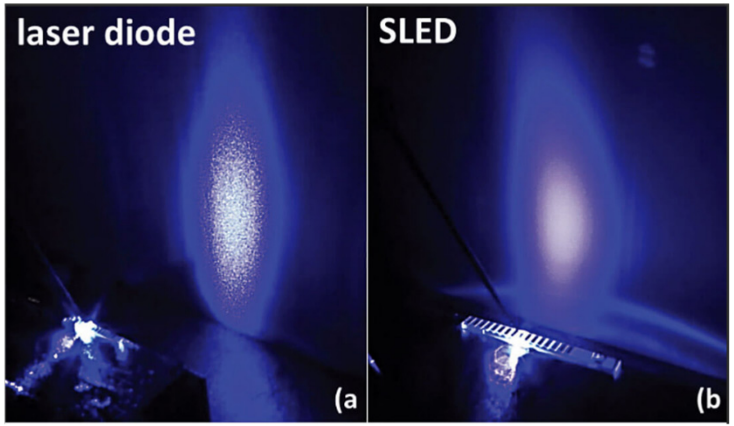
Onset of stimulated emission:

- Sharp emission threshold

- Increase in temporal coherence (decrease in the emission linewidth)

$\Rightarrow$ in the single longitudinal (+ transverse) mode limit, FWHM $\propto P^{-1}$ (modified Schawlow-Townes expression[1] including the linewidth enhancement factor)[2]

- Spatial coherence (beam directionality + speckle pattern due to scattered light from diffuse surfaces)



[1]A. L. Schawlow and C. H. Townes, Phys. Rev. **112**, 1940 (1958). (> 1800 citations)
[2]C. H. Henry, IEEE J. Quantum Electron. **QE-18**, 259 (1982). (> 1940 citations)

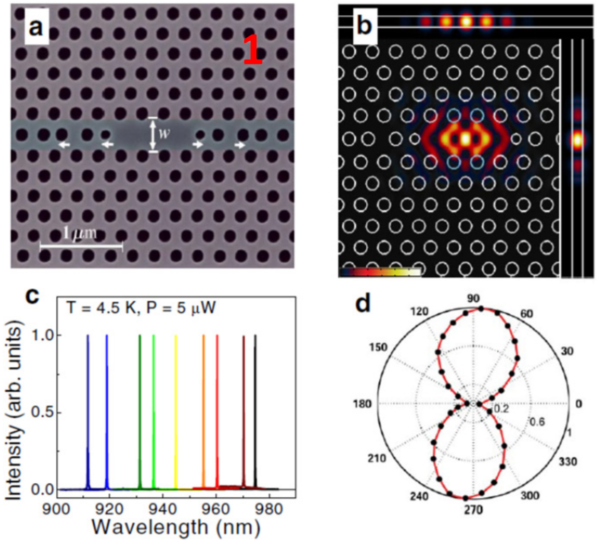
Revisiting semiconductor laser rate equations

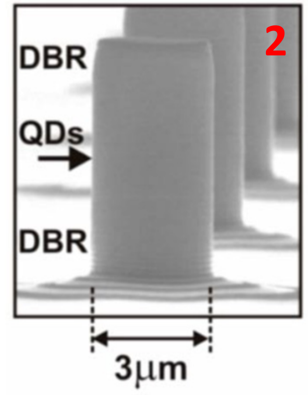
$$\boxed{\begin{aligned} \frac{dn}{dt} &= R_{\mathrm{in}} - \frac{n}{\tau} - v_{\mathrm{g}}\gamma(n)s = R_{\mathrm{in}} - \left(An + Bn^2 + Cn^3\right) - v_{\mathrm{g}}\gamma(n)s \\ \frac{ds}{dt} &= \left[\Gamma v_{\mathrm{g}}\gamma(n) - \frac{1}{\tau_{\mathrm{cav}}}\right]s + \Gamma\beta Bn^2 \end{aligned}}$$

Photon density spontaneously emitted in the lasing mode overlapping with the gain medium

R_{in}	pumping rate per volume unit
$-\frac{n}{\tau}$	carrier density recombination rate
$-v_{\mathrm{g}}\gamma(n)s$	carrier density loss rate due to stimulated photon generation
$\Gamma v_{\mathrm{g}}\gamma(n)s$	stimulated photon density generation rate
$-\frac{s}{\tau_{\mathrm{cav}}}$	net stimulated photon density loss rate due to cavity leakage and waveguide losses
$\Gamma\beta Bn^2$	photon density spontaneously emitted in the lasing mode overlapping with the gain medium
$\Gamma = \frac{V_{\mathrm{e}}}{V_{\mathrm{p}}}$	gain medium - stimulated photon overlap factor $\equiv$ confinement factor
β	spontaneous emission coupling factor $\equiv$ reciprocal number of modes in the bandwidth of the spontaneous emission $\equiv$ fraction of photons of spontaneous origin that is emitted into the lasing mode

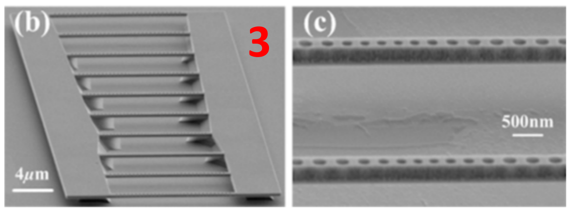
Case of high-β nanolasers





- Various nanocavity geometries with mode volume near the diffraction-limit
- Preferred gain medium: QDs (less sensitive to nonradiative channels) + Purcell effect at low T(K)
- Efficient funneling of spontaneous light emission into the lasing mode

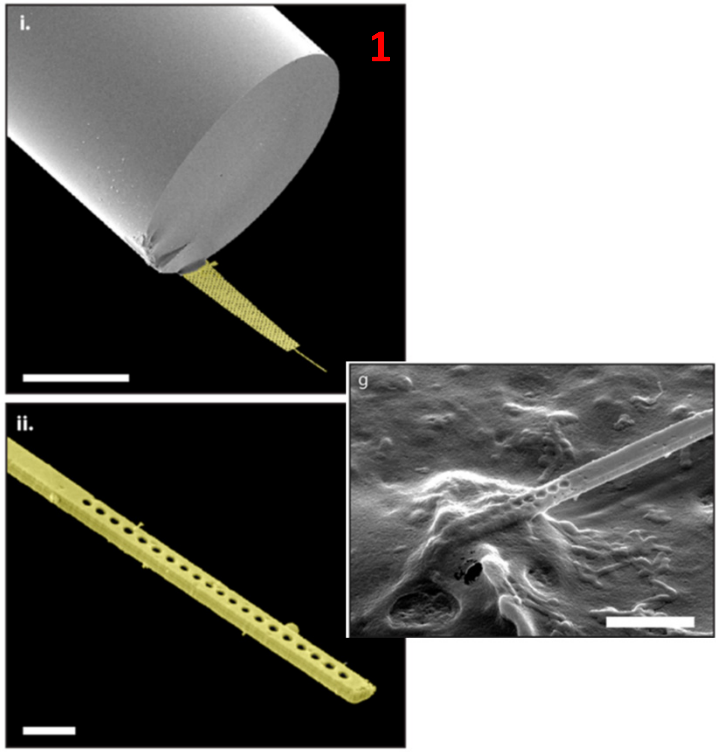
$\Rightarrow$ spontaneous emission coupling factor $\beta > 0.1$ (specific to small mode volume gain media)



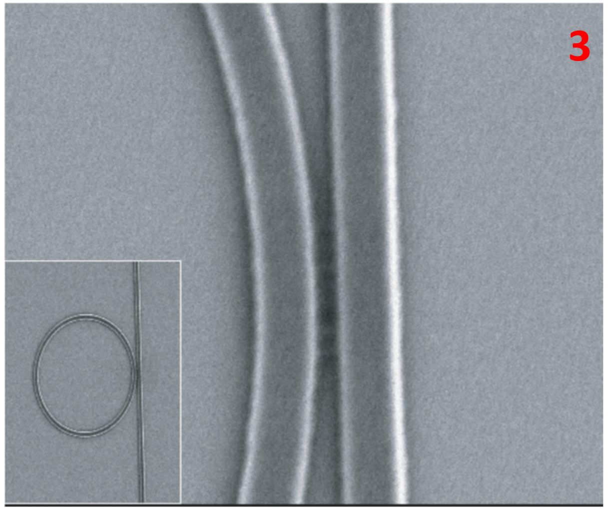
[1]S. Strauf *et al.*, Phys. Rev. Lett. **96**, 127404 (2006). (> 470 citations)
[2]S. M. Ulrich *et al.*, Phys. Rev. Lett. **98**, 043906 (2007). (> 190 citations)
[3]Y. Zhang *et al.*, Appl. Phys. Lett. **97**, 051104 (2010).

High-β nanolasers: applications

Biophotonics



Photonic integrated circuits

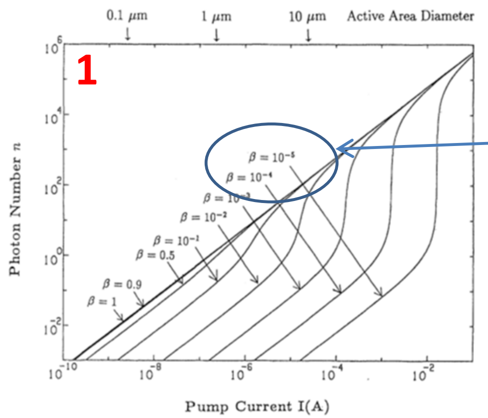


- Optical interconnects for future electronic chips[2]
- Photonic circuits for highly integrated optical communication components[3]

[1]G. Shambat *et al.*, Nano Lett. **13**, 4999 (2013).
[2]D. A. B. Miller, Proc. IEEE **97**, 1166 (2009). (> 1630 citations)
[3]V. R. Almeida *et al.*, Nature **431**, 1081 (2004). (> 1280 citations)

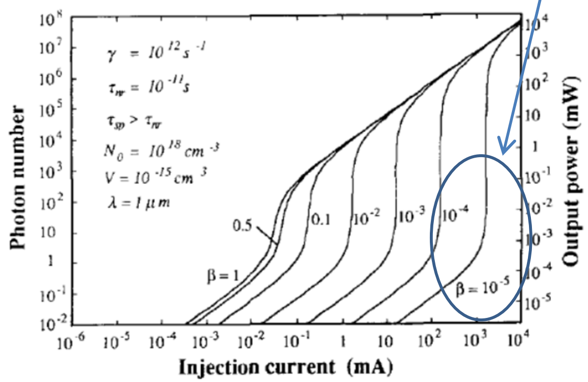
Case of high-β nanolasers

Without nonradiative recombinations

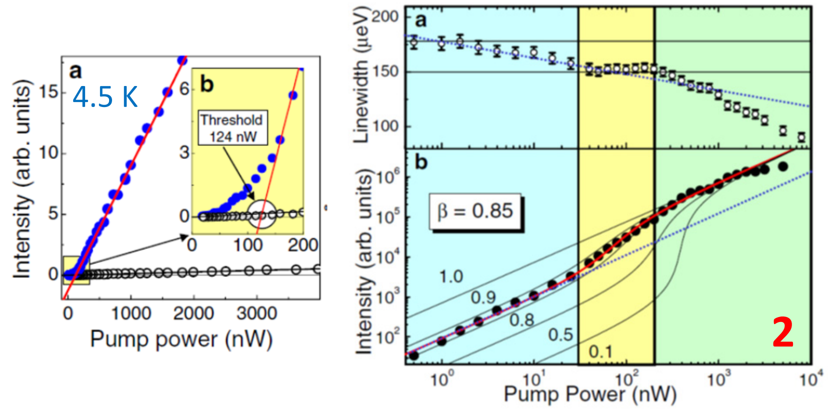


Case of conventional LDs!

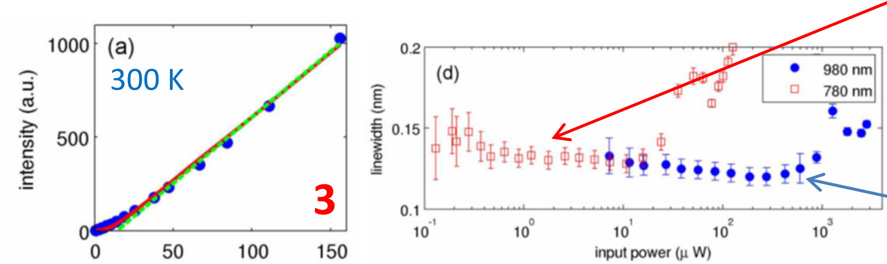
Including nonradiative recombinations



Modified L3 photonic crystal cavity with QD gain medium



GaAs 1D-nanobeam with InAs QDs



Nonresonant pumping

Resonant pumping

[1]Y. Yamamoto *et al.*, Phys. Rev. A **44**, 657 (1991). (> 260 citations)
[2]S. Strauf *et al.*, Phys. Rev. Lett. **96**, 127404 (2006). (> 470 citations)
[3]Y. Gong *et al.*, Opt. Express **18**, 8781 (2010). (> 90 citations)

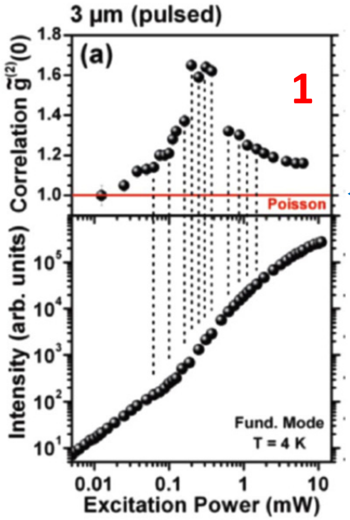
Case of high-β nanolasers

Breakdown of conventional lasing features

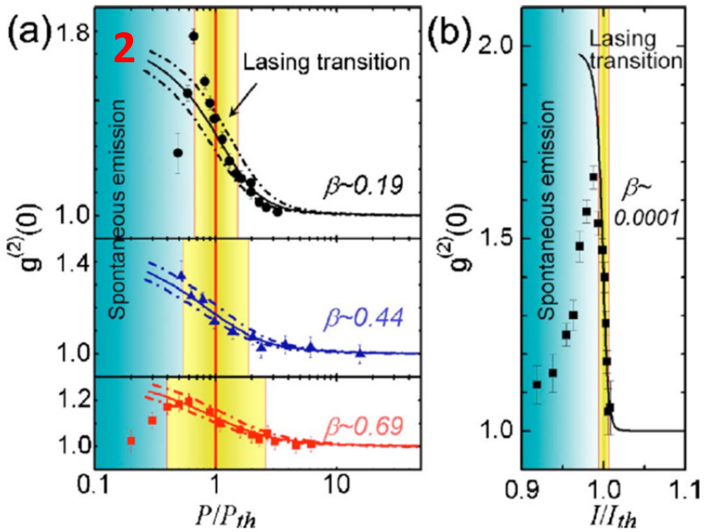
- Smooth transition from spontaneous to stimulated emission regime (vanishing jump in input-output curve)
- Lack of any obvious linewidth narrowing
- Lack of well-defined far-field emission pattern (+ diffraction-limited mode volume leading to emission pattern not purely linked to the onset of coherence)

$\Rightarrow$ **Need for an extra proof of lasing**

2nd-order autocorrelation function measurements



$g^{(2)}(0) = 1$ for a coherent light emitter



$$\tilde{g}^{(2)}(\tau)=\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{+\infty} d\tau' g^{(2)}(\tau-\tau')\exp\left(-\frac{\tau'^2}{2\sigma^2}\right)$$

with $g^{(2)}(\tau)=1+b_0\exp\left(-\frac{2|\tau|}{\tau_c}\right)$

where b_0 is the bunching amplitude,

τ_c is the coherence time,

and $2\sigma=\Delta t_{IRF}$

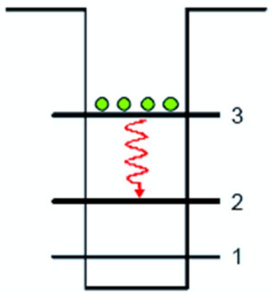
where Δt_{IRF} is the instrument response function

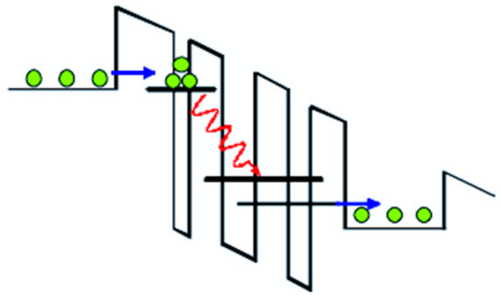
Increasing β coefficient $\Rightarrow$

- Decreasing P_{thr} and blurred lasing transition
- $g^{(2)}(0)$ deviates from 2 below threshold (mostly due to $\Delta t_{IRF} > \tau_c$)
- Slow transition to Poisson limit

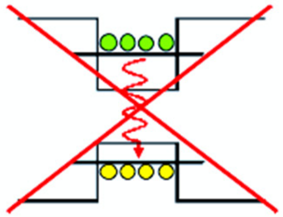
[1] S. M. Ulrich *et al.*, Phys. Rev. Lett. **98**, 043906 (2007). (> 190 citations)
[2] Y.-S. Choi *et al.*, Appl. Phys. Lett. **91**, 031108 (2007).

Quantum cascade lasers[1]





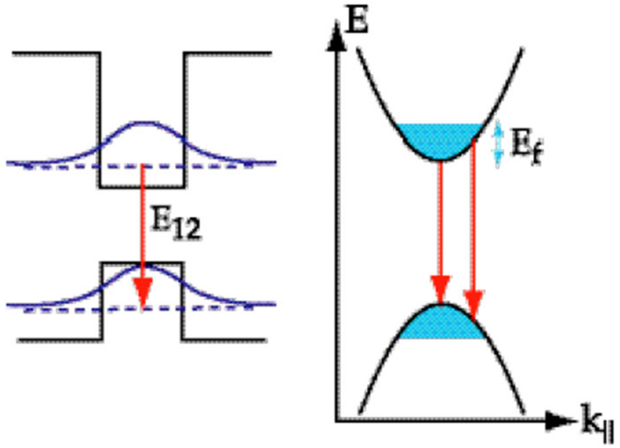
NO ELECTRON - HOLE RECOMBINATION

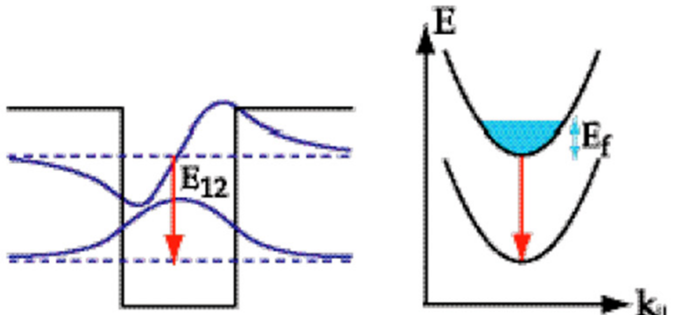


[1]J. Faist *et al.*, Science **264**, 553 (1994). (> 4430 citations)

Quantum cascade lasers

Interband vs intersubband

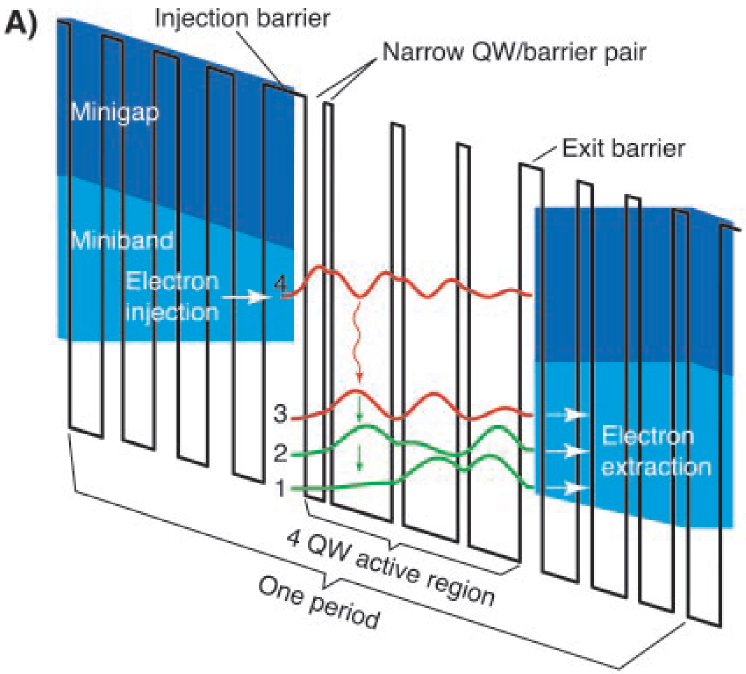


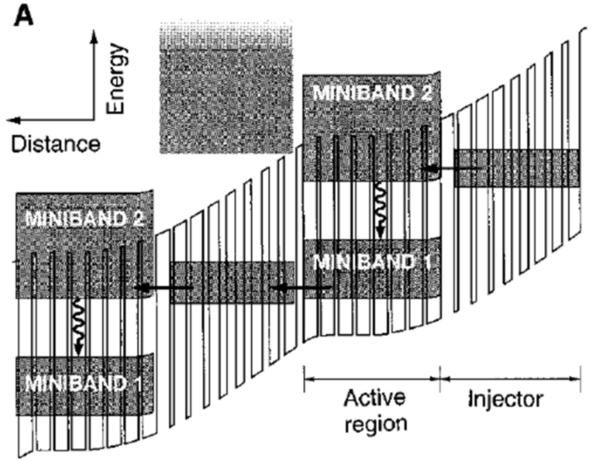


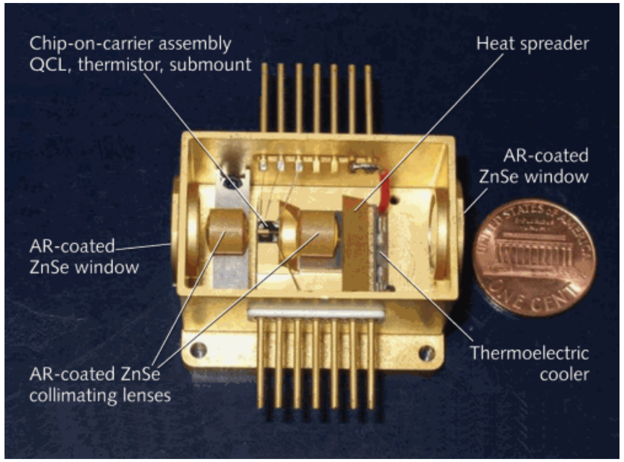
- **Interband transition**
  - bipolar
  - photon energy limited by bandgap E_g of material
  - Telecom, CD, DVD,…

- **Intersubband transition**
  - unipolar, narrow gain
  - photon energy depends on layer thickness and can be tailored

Quantum cascade lasers[1-2]

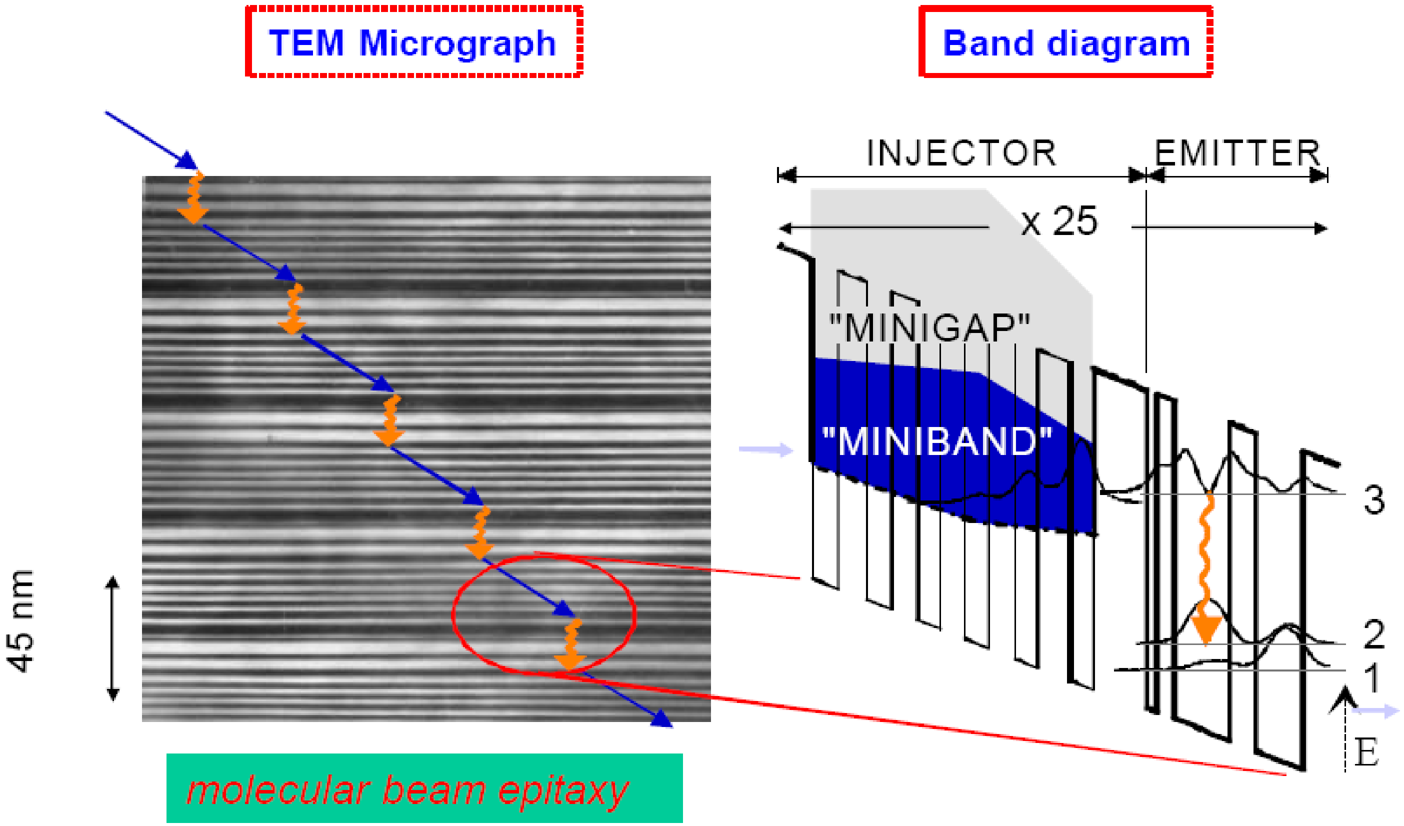




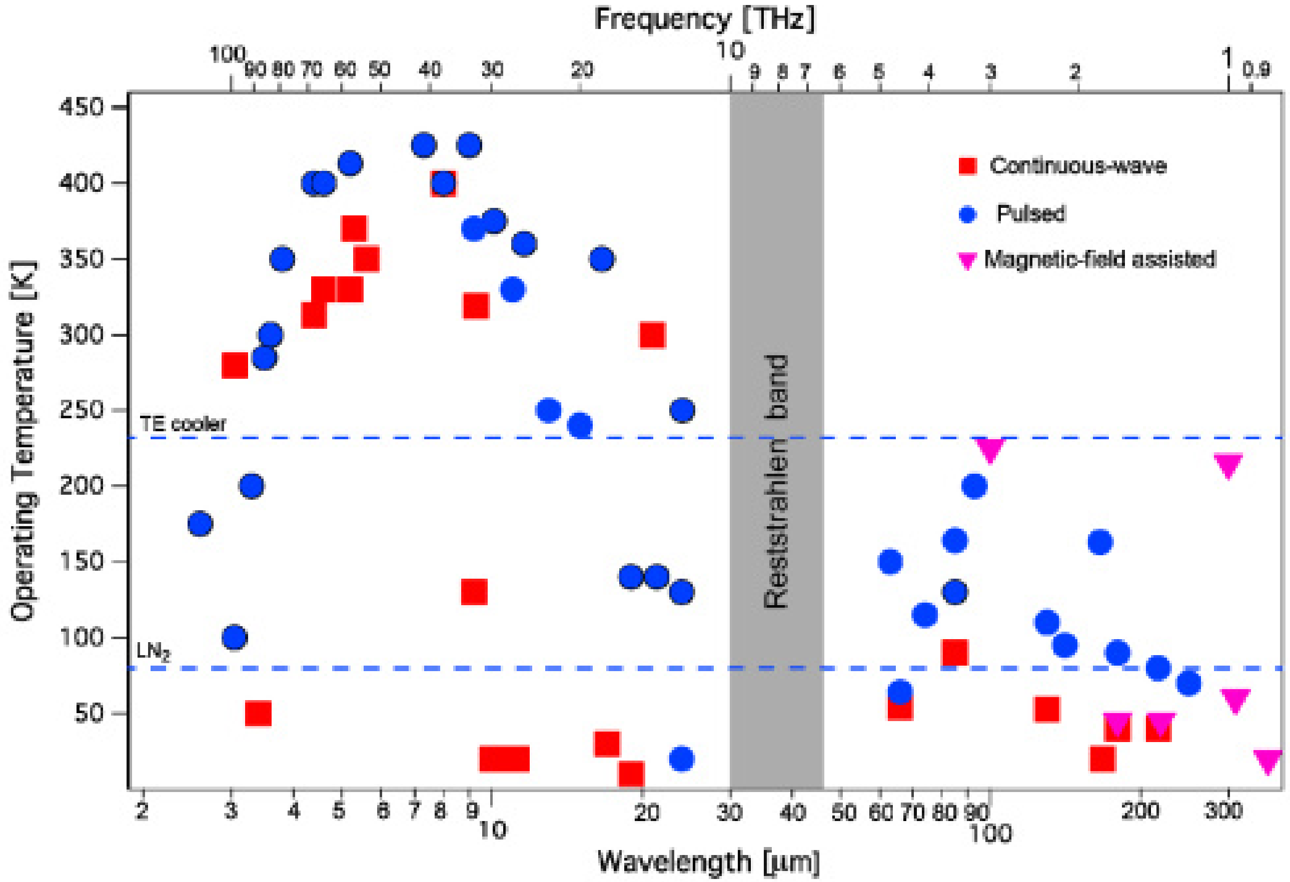


[1]G. Scamarcio *et al.,* Science **276**, 773 (1997) (> 150 citations)
[2]M. Beck *et al.,* Science **295**, 301 (2002). (> 740 citations)

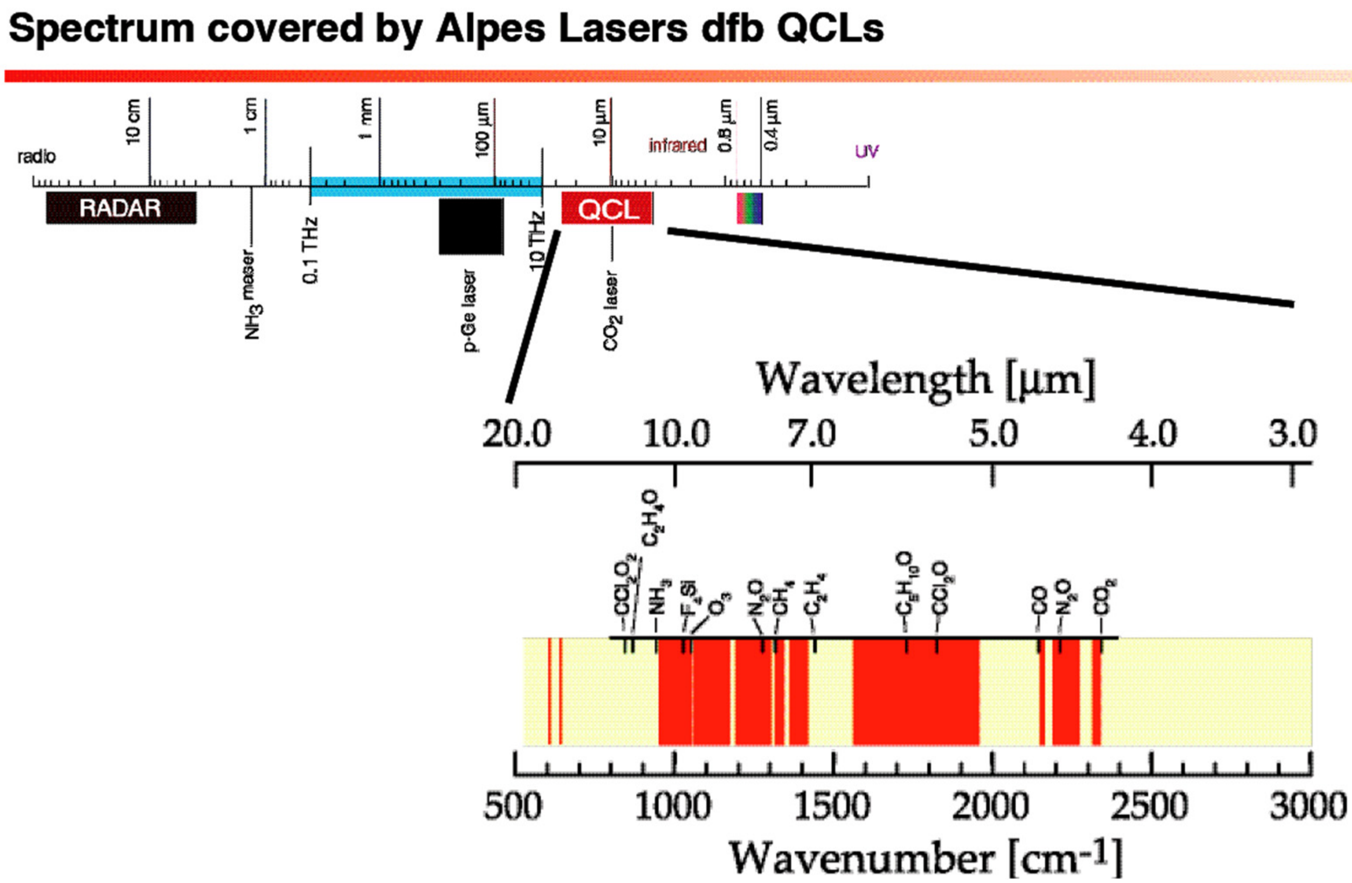
Quantum cascade lasers



Quantum cascade lasers



Quantum cascade lasers



Examination protocol

Examination: 01/07/2025 from 9:15am until 12:15pm in room PH H3 31

- The exam could cover any of the topics addressed during the 14 lectures of this semester and the corresponding series.
- The exam will be a mixture of problem solving (involving basic mathematical calculations) and analysis of figures with a main focus on the underlying physics. Special care will be paid to the quality and the correctness of your explanations and/or arguments.
- The exam shall be written in *readable* English (pay a special attention to your handwriting…).
- Printed versions of the lectures with your personal notes as well as the exercises (+ solutions) are allowed but you cannot bring anything else! Any tool providing access to the Internet is strictly forbidden.
- The reference for all the courses of this semester will be the version of the lectures posted on the Moodle repository at the end of the teaching semester, i.e., the files available <u>from June 2 2025</u>!
- Do not forget to bring a **scientific calculator** and a **ruler** for numerical applications. There won't be any of them on loan!